Lab 13: Chapter 10

Stating Hypotheses **In exercises 1–5, the statement represents a claim. Write down the H_0 and the H_a for each question.**

1. $p < 0.55$

$H_0: p = 0.55$ (or $p \geq 0.55$)

$H_A: p < 0.55$

2. $p > 0.07$

$H_0: p = 0.07$

$H_A: p > 0.07$

3. $p \neq 0.29$

$H_0: p = 0.29$

$H_A: p \neq 0.29$

4. $p < 0.45$

$H_0: p = 0.45$

$H_A: p < 0.45$

5. $\mu > 158$

$H_0: \mu = 158$

$H_A: \mu > 158$

6. Which of the following are legitimate hypotheses?

Which of the 4 statements are statistical hypotheses?

(a) $p = 0.65$ — (a) y

(b) $\hat{p} = 0.60$ — (b) n

(c) $\hat{p} = 0.10$ — (c) n

(d) $p > 3.4$ — (d) y

> because both statements use sample notation and not population notation, these are not statistical hypotheses

7. Which of the following specify legitimate pairs of null and alternative hypotheses?

(a) $H_0 : p = 0.25$ $H_a : p > 0.25$ — (a) yes / legit

(b) $H_0 : \hat{p} = 0.60$ $H_a : \hat{p} > 0.60$ — (b) no / illegit

(c) $H_0 : p > 0.45$ $H_a : p < 0.45$ — (c) no / illegit

(d) $H_0 : p \neq 0.35$ $H_a : p = 0.35$ — (d) no / illegit

(e) $H_0 : p = 0.12$ $H_a : p \neq 0.12$ — (e) yes / legit

Stating the Hypotheses **In Exercises 8–12, write the claim as a mathematical statement. State the null and alternative hypotheses, and identify which represents the claim.**

8. A recent poll (Washington Post – Schar School, Oct. 2019) claimed that a majority of Americans want Trump impeached and removed. Suppose you plan to select a representative sample of 100 students at the college. You will ask each student in the sample if he or she want Trump impeached and removed. You plan to use the resulting data to decide if there is evidence that the majority of students at the school want Trump impeached and removed. What hypotheses should you test?

9. Nearly 60 percent of adult men report drinking in the last month, and approximately 23% of adult men report binge drinking 5 times a month, averaging 8 drinks per binge. Suppose you plan to select a random sample of 100 male students at the college. You will ask each student in the sample questions about their alcohol use, including the question, "How many times per month do you binge drink?" You plan to use the resulting data to decide if there is evidence that the proportion of male students at the college who report binge drinking 5 times a month is under 23%. What hypotheses should you test?

10. Earlier this year, Pew Research Center found that 55% of Americans had a negative impression of socialism, while 42% expressed a positive view. About two-thirds (65%) said they had a positive view of capitalism, and a third viewed it negatively. Suppose you plan to select a representative sample of 100 students at the college. You will ask each student in the sample if he or she has a negative view of capitalism or a positive view of capitalism. You plan to use the resulting data to decide if there is evidence that more than a third of the college's students have a negative view of capitalism. What hypotheses should you test?

11. In a September 6, 2019, Gallup reported that 68% of Americans believe the government is withholding information about UFOs.Suppose you plan to select a representative sample of 100 students at the college. You will ask each student in the sample if he or she has a negative view of capitalism or a positive view of capitalism. You plan to use the resulting data to decide if there is evidence that more than a third of the college's students have a negative view of capitalism. What hypotheses should you test?

12. A new national poll by Quinnipiac University (Oct. 17 – 21, 2019), cited by fox news network, indicates that 55 percent of registered voters support the House Democrats' impeachment investigation into President Trump, with 43 percent opposed. A margin of error of ± 3.1 percentage points was reported. Interpret the meaning of the margin of error in the context of this problem.

Interpreting a Decision **In Exercises 13 and 14, write down the null hypothesis and the alternative hypothesis. If a hypothesis test is performed, how should you interpret a decision that**
(a) rejects the null hypothesis?
(b) fails to reject the null hypothesis?

13. A news poll claims that the majority of Americans want Trump impeached and removed from office.

$H_0: p = 0.5$

$H_A: p > 0.5$ (A majority)

14. According to a 2018 Gallup poll, 5% of U.S. adults consider themselves to be vegetarian. You plan to use sample data to decide if there is evidence that the proportion of adults who consider themselves to be vegetarian is different from 5%.

p = the actual % U.S adults who consider themselves to be vegetarian.

$H_0: p = 0.05$

$H_A: p \neq 0.05$ (p is different from)

Identifying Tests **In Exercises 15–17, determine whether the hypothesis test is left-tailed, right-tailed, or two-tailed.**

15. $H_0: p = 0.25$
$H_a: p > 0.25$

Since H_A has a greater than symbol in it, we have a right-tailed test.

16. $H_0: p = 0.72$
$H_a: p \neq 0.72$

Since H_A has a not equals symbol in it, we have a two-tailed test.

17. $H_0: p = 0.34$
$H_a: p < 0.34$

Since we have a less than symbol in it, we have a left-tailed test.

7b illegit, b.c. the 2 statements use sample notation (i.e, $\hat{p}$)

7c illegit, b.c the symbol used in the null was <u>not</u> $=$, $\leq$, or $\geq$.

7d illegit, same reason as 7c.

(8) claim: $p > 0.5$

Since the claim has the $<$ symbol,

$H_A: p > 0.5$ (claim)

So, $H_0: p = 0.5$ or $p \leq 0.5$

(9) p = the actual percent of male students at the college who report binge drinking 5 times a month

claim: $p < 0.23$

$H_0: p = 0.23$

$H_A: p < 0.23$ (claim)

(10) p = the actual percent of the college's students who have a negative view of capitalism

Claim: $p > \frac{1}{3}$

$H_0: p = \frac{1}{3}$

$H_A: p > \frac{1}{3}$

13a) If we reject the null hypothesis, then we have convincing sample^ evidence ~~of the~~ that the majority of americans ~~data~~ want Trump imp'd & rem'd.

13b) If we fail to reject the null hyp., then we do not have convincing evidence that a majority of Americans want trump imp'd & rem'd.

14a There is convincing sample evidence to support that the actual percent of U.S. Adults who consider themselves to be vegetarian is different from 5%

14b not

18. A Newsweek article titled "America the Ignorant" (www.newsweek.com) described a Gallup poll that asked adult Americans if they believe that there are real witches and warlocks. Suppose that the poll used a random sample of 800 adult Americans and that you want to use the poll data to decide if there is evidence that more than 10% of adult Americans believe in witches and warlocks. Let p be the proportion of all adult Americans who believe in witches and warlocks.

    (a) Describe the shape, center, and spread of the sampling distribution of $\hat{p}$ for random samples of size 800 if the null hypothesis $H_0 : p = 0.10$ is true.

    (b) Would you be surprised to observe a sample proportion of $\hat{p} = 0.16$ for a sample of size 800 if the null hypothesis $H_0 : p = 0.10$ were true? Explain why or why not.

    (c) Would you be surprised to observe a sample proportion of $\hat{p} = 0.11$ for a sample of size 800 if the null hypothesis $H_0 : p = 0.10$ were true? Explain why or why not.

Identifying Errors **In Exercises 18–19, describe type I and type II errors for a hypothesis test of the indicated claim.**

19. A garden hose manufacturer advertises that the mean flow rate of a certain type of hose is 16 gallons per minute.

20. A computer repairer advertises that the mean cost of removing a virus infection is less than $100.

(18)

is described by the standard deviation of the sampling dist of $\hat{p}$, or

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.10(1-0.10)}{800}}$$

$$\approx 0.0106$$

or about 1.06%

(B) The z-score of $\hat{p}$ is

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.16 - 0.10}{0.0106} = \frac{0.06}{0.0106}$$

$$= 5.66$$

st. dev lengths

This places $\hat{p} = 0.16$ in the extreme right-tail of the sampling dist. of $\hat{p}$. Moreover,

(18) p = the actual percent of adult Americans who believe in witches and warlocks.

Claim to test: Is $p > 0.10$??

$H_0: p = 0.10$

$H_A: p > 0.10$

$p_0 = 0.10$

(a) $n \times p_0 = 800(0.10) = 80$ and

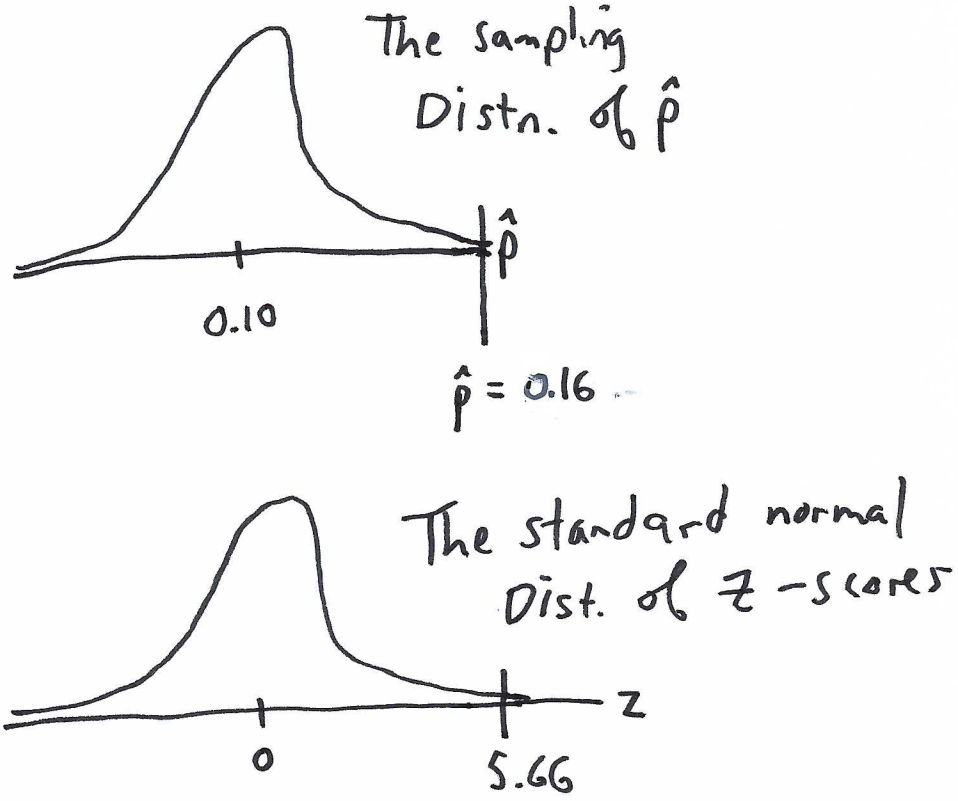
$n \times (1 - p_0) = 800(1 - 0.10) = 720.$

So, at least 80 successes (80 yes responses) and 720 failures (no responses) are expected in a sample of 800.

Therefore, the sampling distn of $\hat{p}$ is approx. normal (bell-shaped), and centered at $\mu_{\hat{p}} = p_0 = 0.10$.

The spread of the sampling distn curve

(18b)



$P(\hat{p} \geq 0.16$, assuming H_0 is correct)

$= P(z \geq 5.66)$

$= \text{normalCDF}(5.66, 10^9, 0, 1) = 7.59 \times 10^{-9}$

≈ 0

So, it would be surprising to get $\hat{p} = 0.16$ from a random sample of $n = 800$, since it is nearly impossible to get $\hat{p} = 0.16$.

(18c) The z-score of $\hat{p} = 0.11$ is

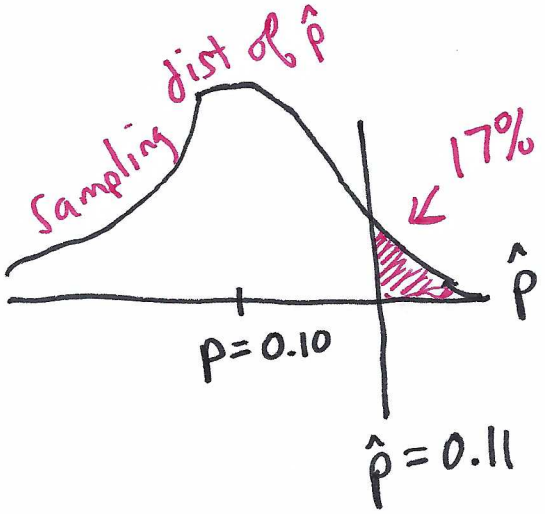
$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.11 - 0.10}{0.0106} = \frac{0.01}{0.0106} = \boxed{0.94}$$

This places $\hat{p} = 0.11$ very close to $p = 0.10$ since $\hat{p} = 0.11$ is less than 1 standard deviation length away from $p = 0.10$. More over,

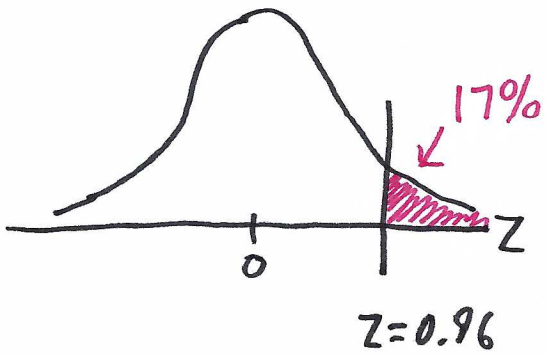
$P(\hat{p} \geq 0.10,$ assuming $p = 0.10)$

$= P(z \geq 0.94) =$ normalCDF$(0.94, 10^9, 0, 1)$

$= 0.1736$ or 17%



and



(18c) Conclusion

I would not be surprised to observe a sample proportion equal to 11% if the actual pop. proportion is equal to 10% since the probability of observing a sample proportion ~~equals~~ equals or is greater than 11% is about 17%. In other words, we expect about 17% of $\hat{p}$ to be greater than or equal to $\hat{p} = 0.11$, and since 17% is not a really low percentage, getting $\hat{p} = 0.11$ would not be unusual or surprising.

(19) $H_0: \eta = 16$ gal./min ← false

$H_A: \eta \neq 16$ gal/min

H_0: the manufacturer's claim

H_A: the avg. flow rate is different from (not) 16 gal/min

Type 1 A type I error occurs when we conclude that the avg flow rate is different from 16 gal/min, when in actuality it is not.

Type II A type II error occurs when we conclude that there is not convincing sample evidence that suggest the flow rate is ~~not~~ different from 16 gal/min., when in it is.

(20) $H_0: \mu = \$100$ (or $\mu \geq \$100$)

$H_A: \mu < \$100$ ←

Type I error A type I error occurs when we conclude there is convincing sample evidence that the actual avg repair cost is less than $100, when in actuality it is not.

Type II error A type II error occurs when we conclude that there is not convincing sample evidence that the avg. repair cost is less than $100, when in actuality it is!

Ex A researcher claims that the actual percentage of New Yorkers that have Covid-19 antibodies in there blood is less than 15% of the population of New Yorkers.

Soln

H_0: $p = 0.15$ (or $p \geq 0.15$)

H_A: $p < 0.15$

Type I A type I error occurs when we conclude that there is convincing sample evidence that the actual percent of New Yorkers who contracted the virus and got better is less than 15% of New Yorkers, when in actuality it is not.

Type II A type II error occurs when we conclude that there is not convincing sample evidence that the actual percent of new yorkers with the antibodies is less than 15%, when in actuality the percent is less 15%.